

Effective potential of the $O(N)$ linear sigma-model at finite temperature

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Abstract. We study the $O(N)$ symmetric linear sigma-model at finite temperature as the low-energy effective models of quantum chromodynamics (QCD) using the Cornwall-Jackiw-Tomboulis (CJT) effective action for composite operators. It has so far been claimed that the Nambu-Goldstone theorem is not satisfied at finite temperature in this framework unless the large- N limit in the $O(N)$ symmetry is taken. We show that this is not the case. The pion is always massless below the critical temperature, if one determines the propagator within the form such that the symmetry of the system is conserved, and defines the pion mass as the curvature of the effective potential. We use a regularization for the CJT effective potential in the Hartree approximation, which is analogous to the renormalization of auxiliary fields. A numerical study of the Schwinger-Dyson equation and the gap equation is carried out including the thermal and quantum loops. We point out a problem in the derivation of the sigma meson mass without quantum correction at finite temperature. A problem about the order of the phase transition in this approach is also discussed.

PACS. 11.30.Rd Chiral symmetries – 11.10.Wx Finite-temperature field theory – 12.39.Fe Chiral Lagrangians

1 Introduction

Chiral symmetry is one of the most important features of low-lying hadron properties. In quantum chromodynamics (QCD) this symmetry is well satisfied in the $SU(2)$ sector due to the light u, d quark masses. Existence of the nearly massless mesons, or pions, means that the chiral symmetry must be spontaneously broken and the Nambu-Goldstone (NG) bosons appear. Spontaneous chiral symmetry breaking (SCSB) is also manifested in the non-degenerate parity doublet of nucleons. So SCSB influences the low-lying hadron spectra significantly. Though such low-energy phenomena of the strong interaction as well as the quark confinement are now widely confirmed in experiments, no analytic investigation based on the first principle, or QCD is carried out due to its non-perturbative nature. So it is still important to study the effective models of QCD in order to understand the nature of hadrons in addition to numerical analyses in the lattice gauge theory.

In increasing temperature, as shown in the lattice QCD calculation, the chiral symmetry is believed to be restored at around $T = 100\text{--}300$ MeV. The study of physics at

finite temperature is very interesting from both theoretical and experimental points of view. The big bang model tells us that a series of phase transitions, which of course include the QCD phase transition, occurred in the early universe. It could also be possible to probe the underlying physics of QCD in laboratory involving relativistic heavy-ion collisions. These experiments are planned in near future and its result is expected to elucidate some important questions like the mechanism of the chiral symmetry restoration and the nature of quark gluon plasma.

Theoretical investigation of the symmetry restoration at finite temperature in terms of field theory was first studied by Kirzhnits and Linde [1]. They observed that spontaneous symmetry breaking (SSB) will be restored at sufficiently high temperatures. It is now well known that the naive effective potential up to the 1-loop level does not work at finite temperature. Weinberg pointed out that at very high temperature, powers of temperature T can compensate for powers of a small coupling constant e , leading to a breakdown of the perturbation expansion [2]. He showed that the leading effect of this sort arises from the $e^2 T^2$ term in the scalar ϕ^4 theory. Dolan and Jackiw and others showed that systematic summation of a certain kind of loop diagrams in this model is needed at finite temperature [3, 4].

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For analysis of the linear sigma-model at finite temperature, the mean-field approximation has mainly been used so far. The mean-field theory is a theory in which the resummation of the diagrams is introduced approximately by hand and has been used by many authors [5–7].

Recently several analyses at finite temperature have been done using an extended effective potential for composite operators introduced by Cornwall, Jackiw, and Tomboulis (CJT) [8]. Contrary to the usual effective action, their effective action depends not only the classical field $\phi_c(x)$ but also on $G(x, y)$. These two quantities are to be realized as the expectation values of a quantum field $\phi(x)$ and the time ordered product of the field operator $T\phi(x)\phi(y)$, respectively. In this case the effective action $\Gamma(\phi_c, G)$ is the generating functional of the two-particle irreducible vacuum graphs. This formalism was originally written at zero temperature but it has been extended for finite temperature in ϕ^4 theory by Amelino-Camelia and Pi [9].

There is an advantage in using the CJT formalism to calculate the effective potential in the Hartree approximation. According to ref. [9], we need to evaluate only one graph that of the “double bubble” instead of summing infinite many “daisy” and “superdaisy” graphs using the usual tree level propagators. These infinite diagrams are incorporated automatically in the effective action if the CJT formalism is used, so the effective potential takes real values at finite temperature. The need of the resummation of this kind of loop diagrams at finite temperature is also discussed in other approaches [10].

Amelino-Camelia *et al.* investigated the $O(N)$ linear sigma-model, which is regarded as the low-energy effective model of the sigma-meson and the pions, at finite temperature with the CJT action [12,13]. It is known, however, that in their approach one encounters a difficulty that the NG theorem appears to be violated in the broken symmetry phase at finite temperature. They concluded that in the CJT formalism the NG theorem is violated in the Hartree(-Fock) approximation, *i.e.*, the case of finite values of N and the so-called superdaisy approximation, while it is satisfied if and only if leading contributions in the $1/N$ expansion are taken. They also discussed the renormalization in the CJT formalism and suggested that a consistent renormalization cannot be performed in the broken phase, since the non-perturbative quantities, or solutions of the Schwinger-Dyson(SD) equations, are included.

In this paper we study two subjects. One is the formulation of the linear sigma-model at finite temperature. We re-analyze the $O(N)$ linear sigma-model at finite temperature in the CJT formalism and show that the NG theorem is always satisfied at any finite values of N in the Hartree approximation. The CJT action does not violate the $O(N)$ symmetry, so that the pions are always massless below the critical temperature if we define the meson masses by the curvature, or the second derivative of the effective potential. On the renormalization, we adopt the method of the renormalization of auxiliary fields, which is regarded as a kind of the regularization in the CJT for-

malism. As shown in the CJT’s paper, the solutions of the SD-equation can be thought as a kind of auxiliary field in the mean-field level. This approach has a clear advantage that both the broken and restored phases can be appropriately regularized in a unified way.

The other is the application of the above formalism to low-energy hadron properties. The linear sigma-model has various merits as an effective model of low-energy hadron dynamics. It can describe SSB as in QCD. Furthermore one can study both the symmetry broken phase and the restored phase if desired. Other effective models such as non-linear sigma-model or chiral perturbation theory treat basically the world with SSB.

The contents of this paper are as follows. In sect. 2, we introduce the CJT effective action for composite operators and explain the difference from the ordinary effective action. In sect. 3, we construct an effective potential for the $O(N)$ symmetric linear sigma-model. Here we also point out inconvenience of the conventional formulation. We show that the NG theorem holds even at finite temperature if one adopts the $O(N)$ symmetric form of the propagator and defines the meson masses as the curvatures of the effective potential. We discuss a regularization of the CJT effective potential in sect. 4. It is based on the renormalization of auxiliary fields and was long ago applied to this model in the large- N limit [14,15]. It can remove divergences in the effective potential both in the symmetry broken and restored phases, while the conventional renormalization can be performed only in the restored phase. Using them, we apply the linear sigma-model to the low-energy mesons in sect. 5. As is well known, the $O(4)$ linear sigma-model can describe the physics of the sigma-meson and the pions in low-energy region. We discuss temperature dependence of physical masses of these mesons and sigma condensate corresponding to the pion decay constant. In the last subsection we also comment on the order of the phase transition and the validity of the approximation used here. Finally summary and conclusion are given in sect. 7.

2 Effective action for composite operators

Effective action for composite operators was originally studied in condensed matter physics [16] and then extended to relativistic field theories [17]. We here use an approach introduced by Cornwall, Jackiw and Tomboulis [8]. This is based on functional methods or the path integral representation for the Green functions.

The effective action for composite operators is a generalization of the conventional effective action and is written by $\Gamma[\phi_c, G]$. This is a functional both of the expectations value of the quantum field $\phi_c(x) = \langle 0|\phi(x)|0\rangle$ and of the propagator $G(x, y) = \langle 0|T\phi(x)\phi(y)|0\rangle$. The c -number function $\phi_c(x)$ is also called a classical field. The variational equations

$$\frac{\delta\Gamma[\phi_c, G]}{\delta\phi_c(x)} = 0, \quad (1)$$

$$\frac{\delta\Gamma[\phi_c, G]}{\delta G(x, y)} = 0, \quad (2)$$

determine ϕ_c and G in the vacuum. Equation (2) is nothing but the SD-equation for the propagator G . Furthermore, we can show that the second variational derivative of $\Gamma[\phi_c, G]$ in G leads to the Bethe-Salpeter equation which describes relativistic bound states. For example, in hadron physics, this is used in order to describe meson states as the bound state of the quark-antiquark pair.

We now describe the series expansion for $\Gamma[\phi, G]$. We introduce the so-called tree level 2-point Green function by

$$\begin{aligned} i\mathcal{D}^{-1}(\phi_c, x, y) &= \frac{\delta^2 S[\phi_c]}{\delta\phi_c(x)\delta\phi_c(y)} \\ &= iD^{-1}(x-y) + \frac{\delta^2 S[\phi_c]_{\text{int}}}{\delta\phi_c(x)\delta\phi_c(y)}. \end{aligned} \quad (3)$$

The required series is then

$$\begin{aligned} \Gamma[\phi_c, G] &= S[\phi_c] + \frac{i}{2}\text{TrLn}G^{-1} \\ &+ \frac{i}{2}\text{Tr}\mathcal{D}^{-1}(\phi_c)G + \Gamma_2[\phi_c, G] + \text{const}, \end{aligned} \quad (4)$$

where the trace, the logarithm and the product $\mathcal{D}^{-1}G$ are taken in the functional sense. The conventional effective action $\Gamma[\phi_c]$ is $\Gamma[\phi_c, G]$ by removing G with the condition (2), *i.e.*,

$$\Gamma[\phi_c] = \Gamma[\phi_c, G_0], \quad (5)$$

$$\frac{\delta\Gamma[\phi_c, G_0]}{\delta G_0(x, y)} = 0. \quad (6)$$

The constant which is independent of ϕ and G is evaluated so that eqs. (5) and (6) are satisfied:

$$\begin{aligned} \Gamma[\phi_c, G] &= S[\phi_c] + \frac{i}{2}\text{TrLn}DG^{-1} \\ &+ \frac{i}{2}\text{Tr}(\mathcal{D}^{-1}(\phi_c)G - 1) + \Gamma_2[\phi_c, G] \end{aligned} \quad (7)$$

This expression is used in some literature. $\Gamma_2[\phi_c, G]$ is given by all the two-particle and higher two-particle irreducible vacuum graphs in a theory which has the vertices determined by the interaction of the action $S_{\text{int}}[\phi, \phi_c]$ and the propagators $G(x, y)$.

When one considers the case of translation-invariant solutions, one sets $\phi_c(x)$ to a constant ϕ_c and takes $G(x, y)$ to be a function only of $x - y$. The series for the effective potential $V(\phi_c, G)$ can be easily obtained from eq. (4):

$$\begin{aligned} V(\phi_c, G) &= V_0(\phi_c) - \frac{i}{2} \int \frac{d^4k}{(2\pi)^4} \ln G^{-1}(k) \\ &- \frac{i}{2} \int \frac{d^4k}{(2\pi)^4} \text{tr}\mathcal{D}^{-1}(\phi_c, k)G(k) + V_2(\phi_c, G) \end{aligned} \quad (8)$$

where $V_0(\phi_c)$ is the tree-level (classical) effective potential,

$$G(k) = \int d^4x e^{ik \cdot (x-y)} G(x-y), \quad (9)$$

$$\mathcal{D}(\phi_c, k) = \int d^4x e^{ik \cdot (x-y)} \mathcal{D}(\phi_c, x-y), \quad (10)$$

and $-V_2(\phi_c, G)$ is the sum of all two- and higher-order loops two-particle irreducible vacuum graphs of the theory with vertices given by $S_{\text{int}}(\phi_c)$ and propagator $G(k)$. The stationary requirements are then

$$\frac{\partial V(\phi_c, G)}{\partial\phi_c} = 0, \quad (11)$$

$$\frac{\partial V(\phi_c, G)}{\partial G(k)} = 0. \quad (12)$$

These equations are the starting point of our discussion.

3 Formulation of the $O(N)$ linear sigma-model

3.1 Effective potential at finite temperature

In this section we construct the effective potential of the $O(N)$ symmetric linear sigma-model using the CJT formalism. In QCD with two-flavor massless quarks, the chiral $SU(2)_L \times SU(2)_R$ symmetry is satisfied in the Lagrangian level. But this symmetry is spontaneously broken to the $SU(2)_V$ symmetry, because the vacuum, *i.e.*, the ground state of the field configuration violates it. We can see this phenomenon in the $O(4)$ linear sigma-model, because the $O(4)$ symmetry has the same algebra as $SU(2) \times SU(2)$. Four fields in the $O(4)$ linear sigma-model are identified with one sigma-meson and three pions when we regard it as an effective model of hadrons. By spontaneous symmetry breaking from $O(4)$ to $O(3)$, the three fields become massless according to the NG theorem and the remaining one is massive. Here we concentrate on the meson fields and see how this model is formulated with the CJT effective action. We set the arbitrary N instead of $N = 4$ at the stage of the formulation for generality. We use the Euclidean metric hereafter because it is convenient when we extend the model to that with the imaginary time formalism at finite temperature.

The Lagrangian density for the $O(N)$ linear sigma-model is given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^a\partial_\mu\phi^a + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{6N}(\phi^2)^2, \quad (13)$$

where $\phi^2 = \phi^a\phi^a$, a runs over 1 to N and repeated indices are summed. By shifting the field as $\phi^a(x) = \phi^a(x) + \phi_c^a(x)$, The tree-level 2-point Green function \mathcal{D} is obtained:

$$\begin{aligned} \mathcal{D}_{ab}^{-1}(\phi_c, x, y) &= \left. \frac{\delta^2 S}{\delta\phi^a\delta\phi^b} \right|_{\phi=\phi_c} = \left[-\partial_\mu\partial_\mu + m^2 + \frac{2\lambda}{3N}\phi_c^2 \right] \\ &\times \delta^{ab}\delta^4(x-y) + \frac{4\lambda}{3N}\phi_c^a\phi_c^b\delta^4(x-y). \end{aligned} \quad (14)$$

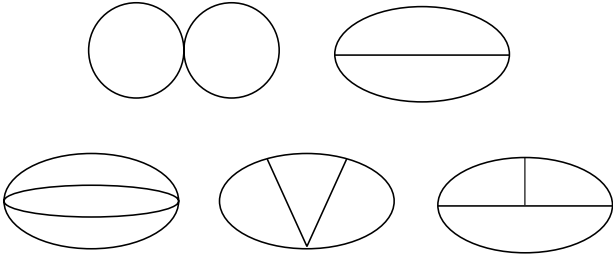


Fig. 1. Two-particle irreducible diagrams contributing to $\Gamma_2[\phi_c, G]$ up to the three-loop level.

In momentum space, it is given by

$$\mathcal{D}_{ab}^{-1}(\phi_c, k) = \left[k^2 + m^2 + \frac{2\lambda}{3N}\phi_c^2 \right] \delta^{ab} + \frac{4\lambda}{3N}\phi_c^a \phi_c^b \quad (15)$$

and the inverse becomes

$$\begin{aligned} \mathcal{D}_{ab}(\phi_c, k) &= \frac{1}{k^2 + m^2 + \frac{2\lambda}{3N}\phi_c^2} \frac{\phi_c^a \phi_c^b}{\phi_c^2} \\ &+ \frac{1}{k^2 + m^2 + \frac{2\lambda}{3N}\phi_c^2} \left(\delta^{ab} - \frac{\phi_c^a \phi_c^b}{\phi_c^2} \right). \end{aligned} \quad (16)$$

The interaction Lagrangian which describes the vertices of the shifted theory is given by

$$\mathcal{L}_{\text{int}}(\phi_c, \phi) = \frac{2\lambda}{3N}\phi^2 \phi_c^a \phi_c^a + \frac{\lambda}{6N}(\phi^2)^2. \quad (17)$$

The diagrams contributing to $\Gamma_2[\phi_c, G]$ are shown in fig. 1. Each line represents the propagator $G_{ab}(x, y)$, and there are two kinds of vertices: a four-point vertex proportional to λ and a three-point vertex, which results from shifting the fields, proportional to $\lambda\phi_c^a(x)$.

If we were computing the ordinary effective action $\Gamma[\phi_c]$, the lines would represent the propagator $\mathcal{D}_{ab}(\phi_c, x, y)$ and there would be additional contributions which are two-particle reducible. These diagrams are shown in fig. 2.

Let us compute the CJT effective action (4). We evaluate $\Gamma[\phi_c, G]$ in the Hartree approximation, *i.e.*, we take into account the “ ∞ ” type diagram only. This diagram is a leading order in $\Gamma_2[\phi_c, G]$ in both the loop expansion and the $1/N$ expansion. The other two-loop diagram, the setting-sun type diagram, which is the next to leading order in the $1/N$ expansion, is quite lengthy to calculate.

In this approximation, Γ_2 is given by

$$\begin{aligned} \Gamma_2(\phi_c, G) &= \frac{\lambda}{6N} \int d^4x (G_{aa}(x, x)G_{bb}(x, x) \\ &+ 2G_{ab}(x, x)G_{ba}(x, x)). \end{aligned} \quad (18)$$

Then the CJT action has the form

$$\begin{aligned} \Gamma[\phi_c, G] &= S[\phi_c] + \frac{1}{2}\text{TrLn}G^{-1} + \frac{1}{2}\text{Tr}(\mathcal{D}^{-1}(\phi_c)G) \\ &+ \frac{\lambda}{6N} \int d^4x (G_{aa}(x, x)G_{bb}(x, x) + 2G_{ab}(x, x)G_{ba}(x, x)), \end{aligned} \quad (19)$$

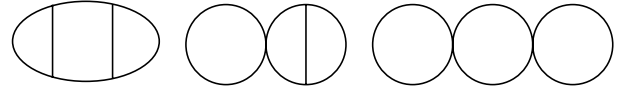


Fig. 2. Two-particle reducible diagrams which do not contribute to $\Gamma_2[\phi_c, G]$ up to the three-loop level.

where $S[\phi_c]$ is the classical action

$$S[\phi_c] = \int d^4x \left(\frac{1}{2}\partial_\mu \phi_c^a \partial_\mu \phi_c^a + \frac{1}{2}m^2 \phi_c^2 + \frac{\lambda}{6N}(\phi_c^2)^2 \right). \quad (20)$$

From this, the effective potential for the composite operators is given by

$$\begin{aligned} V(\phi_c, G) &= V_0[\phi_c] + \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \ln \det G^{-1}(k) \\ &+ \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \text{tr}(\mathcal{D}^{-1}(\phi_c, k)G(k)) \\ &+ \frac{\lambda}{6N} \left[\left\{ \int \frac{d^4k}{(2\pi)^4} G_{aa}(k) \right\}^2 \right. \\ &\left. + 2 \left\{ \int \frac{d^4k}{(2\pi)^4} G_{ab}(k) \right\} \left\{ \int \frac{d^4k}{(2\pi)^4} G_{ba}(k) \right\} \right] \end{aligned} \quad (21)$$

with

$$V_0[\phi_c] = \frac{1}{2}m^2 \phi_c^2 + \frac{\lambda}{6N}(\phi_c^2)^2, \quad (22)$$

where the trace and determinant are taken only in the internal (flavor) space.

If one applies this model to a system at finite temperature, one replaces the loop integral with

$$\int \frac{d^4k}{(2\pi)^4} \longrightarrow T \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \equiv \int_{T,k} \quad (23)$$

in the imaginary time formalism. The sum is taken over the Matsubara frequency. So the starting effective potential we study is

$$\begin{aligned} V(\phi_c, G) &= V_0[\phi_c] + \frac{1}{2} \int_{T,k} \ln \det G^{-1}(k) \\ &+ \frac{1}{2} \int_{T,k} (\mathcal{D}^{-1}(\phi_c, k)G(k)) + \frac{\lambda}{6N} \left[\left\{ \int_{T,k} G_{aa}(k) \right\}^2 \right. \\ &\left. + 2 \left\{ \int_{T,k} G_{ab}(k) \right\} \left\{ \int_{T,k} G_{ba}(k) \right\} \right]. \end{aligned} \quad (24)$$

Minimizing the effective potential with respect to the propagator $G(\phi_c, k)$, we obtain the SD-equation

$$\begin{aligned} G_{ab}^{-1}(p) &= \mathcal{D}_{ab}^{-1}(\phi_c, p) \\ &+ \frac{2\lambda}{3N} \left[\int_{T,k} G_{cc}(k) \delta^{ab} + 2 \int_{T,k} G_{ab}(k) \right]. \end{aligned} \quad (25)$$

The solution $G_0(\phi_c, k)$ of this equation is inserted back into the expression for the effective potential to give the conventional effective potential as a function of ϕ_c .

3.2 How to take the form of the propagator G

We need to determine the form of the SD-equation, G . In case of the $O(N)$ linear sigma-model, it seems to be natural to adopt the following multi-component form:

$$G(k)_{ab}(\phi_c, k) = \delta^{a1} \delta^{b1} \frac{1}{k^2 + M_\sigma^2} + \sum_{j=2}^N \delta^{aj} \delta^{bj} \frac{1}{k^2 + M_\pi^2}. \quad (26)$$

In fact several authors adopt this form [12,13] and in ref. [13] this is called dressed propagator ansatz. They also identify the parameters M_σ and M_π with the masses of the sigma-meson and pions, respectively. It is, however, obvious that this form of G does violate the $O(N)$ symmetry. So the effective potential after substituting the solution G_0 also violates the $O(N)$ symmetry. As discussed below, however, one can derive the correct answer in this approach if one examines the transformation property of M_σ and M_π carefully in order to restore the $O(N)$ symmetry. We calculate the meson masses by using the definitions (38) and (39) from the effective potential directly. In this case it is convenient and transparent to choose the form of G not to break the symmetry. Here we use the following form of G from the analogy of \mathcal{D} :

$$G_{ab}^{-1}(\phi_c, k) = (k^2 + \alpha(\phi_c^2))\delta^{ab} + \beta(\phi_c^2) \frac{\phi_c^a \phi_c^b}{\phi_c^2} \quad (27)$$

where $\alpha(\phi_c^2)$ and $\beta(\phi_c^2)$ are unknown functions which are determined by solving the SD-equation. We assume that α and β are independent of the external momentum and also that no more parameters are needed because the Hartree approximation is employed.

Substituting eq. (27) into the CJT effective potential (8), we obtain

$$\begin{aligned} V(\phi_c, G(\phi_c, k)) &= \frac{m^2}{2} \phi_c^2 + \frac{\lambda}{6N} (\phi_c^2)^2 + \frac{1}{2} Q[\alpha + \beta] + \frac{N-1}{2} Q[\alpha] \\ &+ \frac{1}{2} \left(m^2 + \frac{2\lambda}{N} \phi_c^2 - (\alpha + \beta) \right) P[\alpha + \beta] \\ &+ \frac{N-1}{2} \left(m^2 + \frac{2\lambda}{3N} \phi_c^2 - \alpha \right) P[\alpha] + \frac{\lambda}{6N} [3P[\alpha + \beta]^2 \\ &+ (N^2 - 1)P[\alpha]^2 + 2(N-1)P[\alpha + \beta]P[\alpha]] , \quad (28) \end{aligned}$$

where we introduce

$$P[x] = \int_{T,k} \frac{1}{k^2 + x} , \quad (29)$$

$$Q[x] = \int_{T,k} \ln(k^2 + x) , \quad (30)$$

for simplicity. This effective potential depends only on $\phi_c^2 \equiv \phi_c^a \phi_c^a$ and therefore is $O(N)$ symmetric obviously.

The functions α and β are determined from the following SD-equations:

$$\begin{aligned} \alpha + \beta &= m^2 + \frac{2\lambda}{N} \phi_c^2 + \frac{2\lambda}{N} \\ &\times P[\alpha + \beta] + \frac{2\lambda}{3} \frac{N-1}{N} P[\alpha] , \quad (31) \end{aligned}$$

$$\begin{aligned} \alpha &= m^2 + \frac{2\lambda}{3N} \phi_c^2 + \frac{2\lambda}{3N} \\ &\times P[\alpha + \beta] + \frac{2\lambda}{3} \frac{N+1}{N} P[\alpha] . \quad (32) \end{aligned}$$

On the other hand, when we identify the components of G_{ab} with the propagators of the sigma and pions, *i.e.*, $G_{11} = G_\sigma$, $G_{22} = G_{33} = \dots = G_{NN} = G_\pi$, we obtain

$$\begin{aligned} G_\sigma^{-1}(k) &= k^2 + m^2 + \frac{2\lambda}{N} \sigma_c^2 + \frac{2\lambda}{3} \frac{N-1}{N} \pi_c^2 \\ &+ \frac{2\lambda}{3N} \int \frac{d^4 k}{(2\pi)^4} (3G_\sigma(k) + (N-1)G_\pi(k)) , \quad (33) \end{aligned}$$

$$\begin{aligned} G_\pi^{-1}(k) &= k^2 + m^2 + \frac{2\lambda}{3N} \sigma_c^2 + \frac{2\lambda}{3} \frac{N+1}{N} \pi_c^2 \\ &+ \frac{2\lambda}{3N} \int \frac{d^4 k}{(2\pi)^4} (G_\sigma(k) + (N+1)G_\pi(k)) , \quad (34) \end{aligned}$$

with $\sigma_c = \phi_c^1$, $\pi_c = \phi_c^2 = \phi_c^3 = \dots = \phi_c^N$. Equation (34) is clearly not the $O(N)$ covariant form. After the condensation, $\sigma_c = \sigma_0 \neq 0$, $\pi_c = 0$, these eqs. (33) and (34) are identical to eqs. (31) and (32), respectively, by identifying $G_\sigma^{-1}(k)$ with $k^2 + \alpha + \beta$ and $G_\pi^{-1}(k)$ with $k^2 + \alpha$. Therefore when we derive the SD-equations for α and β , which correspond to M_σ and M_π in the dressed propagator ansatz, both approaches lead to the same equations.

The dressed propagator ansatz means that the $O(N)$ symmetry is violated by putting $\sigma_c = \sigma_0 \neq 0$, $\pi_c = 0$ from the beginning. But it is known that when one replaces σ_0^2 by ϕ_c^2 after all calculation, the resultant equations restore the $O(N)$ symmetry. This is one of simple calculation methods.

The reason why we adopt the $O(N)$ symmetric two-point function (27) is as follows. As be seen later, we get the physical meson masses as the second derivative of the effective potential. In the way of the calculation, we need to obtain the first derivative of the effective potential before the condensation, *i.e.*, the $O(N)$ symmetric form. Even if one employed the dressed propagator ansatz and replaced σ_0^2 by ϕ_c^2 after calculation, one might lead to the right equations by a careful examination. In this case, however, M_σ and M_π also look like the $O(N)$ violated form and it is not clear how to “restore” them into the symmetric form. So it is straightforward to employ the $O(N)$ symmetric form (27).

The gap equation for the sigma condensate is given by

$$\begin{aligned} \left. \frac{dV(\phi_c, G(\phi_c))}{d\sigma_c} \right|_{\sigma_c=\sigma_0, \pi_c=0} &= \\ \left(\frac{\partial V(\phi_c, G(\phi_c))}{\partial \sigma_c} + \frac{\partial V(\phi_c, G(\phi_c))}{\partial G} \frac{dG(\phi_c)}{d\sigma_c} \right) \Big|_{\sigma_c=\sigma_0, \pi_c=0} &= \\ &= \left. \frac{\partial V(\phi_c, G(\phi_c))}{\partial \sigma_c} \right|_{\sigma_c=\sigma_0, \pi_c=0} \\ &= 0 \end{aligned} \quad (35)$$

which gives

$$\sigma_0 \left[m^2 + \frac{2\lambda}{3N} \sigma_0 + \frac{2\lambda}{N} P[\alpha + \beta] + \frac{2\lambda}{3} \frac{N-1}{N} P[\alpha] \right] = 0 \quad (36)$$

This equation reduces to the very simple form when the SD-equations are used again.

$$\sigma_0^2 \left[\alpha + \beta - \frac{4\lambda}{3N} \sigma_0^2 \right] = 0. \quad (37)$$

Here we need some case for the meson masses. The question is whether M_σ and M_π , which are the parameters that appear in the SD-equation, can be identified as the physical masses of the sigma mesons and pions. In fact, it is more appropriate to define the masses by the curvatures of the effective potential around its minimum, namely,

$$m_\sigma^2 \equiv \left. \frac{d^2 V}{d\sigma_c^2} \right|_{\sigma_c=\sigma_0, \pi_c=0}, \quad (38)$$

$$m_\pi^2 \equiv \left. \frac{d^2 V}{d\pi_c^2} \right|_{\sigma_c=\sigma_0, \pi_c=0}, \quad (39)$$

rather than M_σ and M_π . The quantities M_σ and M_π are merely regarded as variational parameters which are determined from the SD-equation. In fact if higher-order loop corrections are taken into account in Γ_2 in the CJT action, the solutions of the SD-equation, M_σ and M_π , become momentum-dependent and then cannot be simply identified with the physical meson masses. Of course, the masses defined by eqs. (38) and (39) are also approximate values unless all the loop contributions are taken into account. But we see that the NG theorem is always satisfied because the $O(N)$ symmetry is satisfied at any order of expansion. This way of defining the masses in the effective potential formalism is also seen in ref. [7, 18]¹. The differences between two definitions come from the truncation of the loop expansion of the CJT action. Since the physical mass is defined by the pole of the two-point function, if one

¹ In [18], the dressed propagator ansatz (26) are used. In this case even if one defines the meson masses by the second derivatives of the effective potential, the NG theorem is violated in the CJT formalism because of the violation of the $O(N)$ symmetry in the effective potential. Final results are correct in that paper, but a careful examination is needed in this approach as discussed previously.

can calculate the effective potential up to all-order, both definitions might give the same mass in the massless pion sector. Our definition of the mass is based on the approximation that respects the $O(N)$ symmetry and therefore the NG-theorem is satisfied at each loop diagram of Γ_2 in the CJT action.

We evaluate the meson masses according to the definitions (38) and (39). The results are

$$m_\sigma^2 = 2\sigma_0^2 \left(\left. \frac{d(\alpha + \beta)}{d(\phi_c^2)} \right|_{\sigma_c=\sigma_0, \pi_c=0} - \frac{4\lambda}{3N} \right), \quad (40)$$

$$m_\pi^2 = 0, \quad (41)$$

in the broken symmetry phase. The pion mass vanishes as expected. The derivative term in the sigma-meson mass is transformed into

$$\begin{aligned} \frac{d(\alpha + \beta)}{d(\phi_c^2)} &= (18\lambda N + 8\lambda^2(N + 2)P_2[\alpha]) \\ &\times (9N^2 + 6\lambda N(N + 1)P_2[\alpha] + 18\lambda N P_2[\alpha + \beta] \\ &+ 8(N + 2)\lambda^2 P_2[\alpha]P_2[\alpha + \beta])^{-1} \end{aligned} \quad (42)$$

with

$$P_2[x] \equiv \sum_{T,k} \frac{1}{(k^2 + x)^2} \quad (43)$$

by the SD-equations.

We discuss the treatment of these equations in a phenomenological application in sect. 5 together with numerical calculation.

4 Regularization

Renormalization of the CJT effective action in the linear sigma-model is discussed in ref. [12] in the past. In that paper, divergent integrals are regularized with a cut-off and the renormalized mass and coupling are defined in order to absorb the divergent part. This renormalization program was performed in the large- N limit [14], where all the divergent terms are absorbed into the bare quantities. However, if one applies this to the present case, one sees that not all the divergences can be removed away. Although all the divergences can be absorbed by the renormalization in the symmetry restored phase, if we use it in the broken phase, all the terms in the SD-equation and the effective potential are expressed not only by the renormalized quantities but bare quantities remain in several terms. Essentially the same situation is encountered in the mean field approach [7].

Under these circumstances, Amelino-Camelia makes the following arguments. If we are interested in the possibility of renormalization, we drop the λ term in the infinite limit of the cut-off. We then get, however, $M_\sigma = M_\pi$. He also says that if one treats the model as the low-energy effective model, the cut-off Λ should be a large finite value

which is smaller than the Landau pole. He concludes that M_π vanishes at $\lambda \rightarrow 0$ but remains finite for the finite value of the cut-off.

We agree that the $O(N)$ linear sigma-model in four dimensions may be trivial if the full quantum effects are included. Our analysis is, however, basically one-loop level and therefore semi-classical. So we consider another way of the renormalization which removes divergences order by order both in the symmetry restored phase and broken phase.

Recently a regularization scheme, which is similar to the Pauli-Villars regularization, is used in the CJT action [20]. We here use another regularization scheme based on auxiliary field method with the dimensional regularization. In the original CJT's paper [8], it is reported that the effective potential in the large- N limit in the CJT formalism is the same form obtained from the auxiliary field method [14]. Since the solutions of the SD-equation, α and β , are momentum independent here, we can regard them as an auxiliary field. Then one can regularize the CJT effective potential with the auxiliary field method following some literature [14, 15].

Following the above argument, we first rewrite the $O(N)$ symmetric CJT effective potential (28) using the SD-equations (31) and (32)

$$\begin{aligned} V(\phi_c, \alpha, \beta) &= \frac{1}{2}(\alpha + \beta)\phi_c^2 - \frac{\lambda}{3N}\phi_c^4 \\ &+ \frac{3N^2}{4\lambda(N+2)}\alpha m^2 + \frac{3N}{4\lambda(N+2)}\beta m^2 - \frac{3N}{16\lambda(N+2)} \\ &\times \{(2N-3)\alpha^2 + 6\alpha\beta + (N+2)\beta^2 + 2Nm^4\} \\ &+ \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \ln(k^2 + \alpha + \beta) + \frac{N-1}{2} \int \frac{d^4k}{(2\pi)^4} \ln(k^2 + \alpha). \end{aligned} \quad (44)$$

Throughout this section, we ignore the temperature-dependent terms, since they are separable and all finite. This means that the loop integrals in the equations are simply four-momentum integrals. We consider that the extension to the system at finite temperature should be carried out for the well-defined effective potential which has no divergence after regularization. Once the regularization is carried out at $T = 0$, it is straightforward to extend it to the system at finite temperature since the temperature dependent terms all converge in the UV limit due to the Boltzmann factor.

We add four counter terms in order to cancel the divergence, which is analogous to ref. [14, 15],

$$A\alpha + B\beta + C\alpha^2 + D\beta^2 + E\alpha\beta, \quad (45)$$

where A, B, C, D and E are ‘‘renormalization’’ constants to adjust the divergences and become

$$\begin{aligned} A &= B = 0, \\ C &= ND = 2NE = \frac{N}{64\pi^2} \left(\frac{2}{4-d} - \gamma_E + \ln(4\pi) \right), \end{aligned} \quad (46)$$

if the $\overline{\text{MS}}$ scheme of dimensional regularization is used. These correspond to the following ‘‘renormalization’’ con-

ditions:

$$\left. \frac{dV}{d\alpha} \right|_{\sigma_c^2=0, \alpha=\beta=0} = \frac{3N^2 m^2}{4\lambda(N+2)}, \quad (47)$$

$$\left. \frac{dV}{d\beta} \right|_{\sigma_c^2=0, \alpha=\beta=0} = \frac{3Nm^2}{4\lambda(N+2)}, \quad (48)$$

$$\left. \frac{d^2V}{d\alpha^2} \right|_{\sigma_c^2=0, \alpha=0, \beta=\mu^2} = -\frac{3N(2N-3)}{8\lambda(N+2)}, \quad (49)$$

$$\left. \frac{d^2V}{d\beta^2} \right|_{\sigma_c^2=0, \alpha=0, \beta=\mu^2} = -\frac{3N}{8\lambda}, \quad (50)$$

$$\left. \frac{d^2V}{d\alpha d\beta} \right|_{\sigma_c^2=0, \alpha=0, \beta=\mu^2} = -\frac{9N}{4\lambda(N+2)}, \quad (51)$$

respectively. Therefore the regularized effective potential is given by

$$\begin{aligned} V(\phi_c, \alpha, \beta) &= \frac{1}{2}(\alpha + \beta)\phi_c^2 - \frac{\lambda}{3N}\phi_c^4 \\ &+ \frac{3N^2}{4\lambda(N+2)}\alpha m^2 + \frac{3N}{4\lambda(N+2)}\beta m^2 - \frac{3N}{16\lambda(N+2)} \\ &\times \{(2N-3)\alpha^2 + 6\alpha\beta + (N+2)\beta^2 + 2Nm^4\} + \frac{(\alpha+\beta)^2}{64\pi^2} \\ &\times \left(\ln \frac{\alpha + \beta}{\mu^2} - \frac{3}{2} \right) + \frac{(N-1)\alpha^2}{64\pi^2} \left(\ln \frac{\alpha}{\mu^2} - \frac{3}{2} \right). \end{aligned} \quad (52)$$

The regularized SD-equations are the same form as eqs. (31), (32) except for the divergent terms:

$$\begin{aligned} \alpha + \beta &= m^2 + \frac{2\lambda}{N}\phi_c^2 + \frac{2\lambda}{N} \frac{\alpha + \beta}{16\pi^2} \left(\ln \frac{\alpha + \beta}{\mu^2} - 1 \right) \\ &+ \frac{2\lambda}{3} \frac{N-1}{N} \frac{\alpha}{16\pi^2} \left(\ln \frac{\alpha}{\mu^2} - 1 \right), \end{aligned} \quad (53)$$

$$\begin{aligned} \alpha &= m^2 + \frac{2\lambda}{3N}\phi_c^2 + \frac{2\lambda}{3N} \frac{\alpha + \beta}{16\pi^2} \left(\ln \frac{\alpha + \beta}{\mu^2} - 1 \right) \\ &+ \frac{2\lambda}{3} \frac{N+1}{N} \frac{\alpha}{16\pi^2} \left(\ln \frac{\alpha}{\mu^2} - 1 \right). \end{aligned} \quad (54)$$

The gap equation for the sigma condensate and the formulae of the meson masses are the same form as eqs. (37), (40) and (41). They are all finite because α and β are finite. Since α and β are not the auxiliary fields in the CJT formalism, the above method is a kind of regularization which is analogous to renormalization of auxiliary fields. However, in the large- N limit, the CJT formalism is equivalent to the auxiliary field method and above α and β is identical to the corresponding auxiliary fields [14]. In fact one can show that their results are identical to ours in the large N limit, *i.e.*, β vanishes and the ‘‘renormalization’’ conditions (47) and (49) correspond to eqs. (2.5) and (2.6) in ref. [15], respectively. Thus we think that it is reasonable to apply it to the present CJT formalism.

4.1 Summary of the formulation

We summarize the results of this section. First we formulate the $O(N)$ linear sigma-model with the CJT effective action. The Hartree-Fock approximation is used, which corresponds to incorporating only the double bubble diagram in the two-loop level in the CJT formalism. We adopt the $O(N)$ symmetric form of the propagator G , otherwise the NG theorem is violated. On the renormalization, we use the method of the auxiliary field. One can remove all the divergence in the effective potential even in the broken phase in this method. The effective potential is then eq. (52) plus the thermal effects:

$$\begin{aligned} V(\phi_c, \alpha, \beta) = & \frac{1}{2}(\alpha + \beta)\phi_c^2 - \frac{\lambda}{3N}\phi_c^4 \\ & + \frac{3N^2}{4\lambda(N+2)}\alpha m^2 + \frac{3N}{4\lambda(N+2)}\beta m^2 - \frac{3N}{16\lambda(N+2)} \\ & \times \{(2N-3)\alpha^2 + 6\alpha\beta + (N+2)\beta^2 + 2Nm^4\} \\ & + \frac{(\alpha + \beta)^2}{64\pi^2} \left(\ln \frac{\alpha + \beta}{\mu^2} - \frac{3}{2} \right) + \frac{(N-1)\alpha^2}{64\pi^2} \\ & \times \left(\ln \frac{\alpha}{\mu^2} - \frac{3}{2} \right) + \frac{1}{2}Q_T[\alpha + \beta] + \frac{N-1}{2}Q_T[\alpha] \end{aligned} \quad (55)$$

with

$$Q_T[x] = \int \frac{d^3k}{(2\pi)^3} \frac{2}{\beta} \ln(1 - e^{-\beta\omega_k}),$$

$$\omega_k + \sqrt{\mathbf{k}^2 + x} \quad (56)$$

α and β are determined by the SD-equations

$$\begin{aligned} \alpha + \beta = & m^2 + \frac{2\lambda}{N}\phi_c^2 + \frac{2\lambda}{N}P_T[\alpha + \beta] \\ & + \frac{2\lambda}{3} \frac{N-1}{N} P_T[\alpha] + \frac{2\lambda}{N} \frac{\alpha + \beta}{16\pi^2} \left(\ln \frac{\alpha + \beta}{\mu^2} - 1 \right) \\ & + \frac{2\lambda}{3} \frac{N-1}{N} \frac{\alpha}{16\pi^2} \left(\ln \frac{\alpha}{\mu^2} - 1 \right), \end{aligned} \quad (57)$$

$$\begin{aligned} \alpha = & m^2 + \frac{2\lambda}{3N}\phi_c^2 + \frac{2\lambda}{3N}P_T[\alpha + \beta] \\ & + \frac{2\lambda}{3} \frac{N+1}{N} P_T[\alpha] + \frac{2\lambda}{3N} \frac{\alpha + \beta}{16\pi^2} \left(\ln \frac{\alpha + \beta}{\mu^2} - 1 \right) \\ & + \frac{2\lambda}{3} \frac{N+1}{N} \frac{\alpha}{16\pi^2} \left(\ln \frac{\alpha}{\mu^2} - 1 \right), \end{aligned} \quad (58)$$

with

$$P_T[x] = \int \frac{d^3k}{(2\pi)^3} \frac{n_B(\omega_k)}{\omega_k}. \quad (59)$$

The sigma condensate is given by eq. (37) which is finite since $\alpha + \beta$ is finite in this case.

The meson masses are also given by eqs. (40) and (41). The pion mass is of course massless in the broken symmetry phase. In the equation of the sigma meson mass, $P_2[m]$ factor is replaced by

$$P_2[x] \rightarrow P_{2,T}[x] - \frac{1}{16\pi^2} \ln \frac{x}{\mu^2}, \quad (60)$$

$$P_{2,T}[x] = \frac{\beta}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_k^2} \left[\left(1 + \frac{1}{\beta\omega_k} \right) n_B(\omega_k) + n_B(\omega_k)^2 \right] \quad (61)$$

in the \overline{MS} scheme of the dimensional regularization.

5 An application to the low-energy mesons

In this and the following sections, we apply the CJT formalism introduced in sect. 3 to systems of low-energy hadrons. We include both the thermal and quantum corrections in numerical calculation.

5.1 Formulation of the explicitly chiral symmetry breaking

First we formulate a system with a sigma-meson and pions. Since the real pions have masses because of the small but non-zero u, d quark masses, we introduce a term which explicitly breaks chiral symmetry in the Lagrangian:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi^a \partial_\mu \phi^a + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{6N} (\phi^2)^2 - c\sigma, \quad (62)$$

with $\phi^1 = \sigma$. Since the two-point function is obtained from the second-derivative of the Lagrangian, this term does not affect it and the SD-equations for $\alpha + \beta$ and α are the same forms as eqs. (57) and (58). Thus the effective potential is the same as eq. (55) except for the $c\sigma$ term:

$$\begin{aligned} V(\phi_c, \alpha, \beta) = & \frac{1}{2}(\alpha + \beta)\phi_c^2 - \frac{\lambda}{3N}\phi_c^4 \\ & + \frac{3N^2}{4\lambda(N+2)}\alpha m^2 + \frac{3N}{4\lambda(N+2)}\beta m^2 - \frac{3N}{16\lambda(N+2)} \\ & \times \{(2N-3)\alpha^2 + 6\alpha\beta + (N+2)\beta^2 + 2Nm^4\} \\ & + \frac{1}{2}Q_T[\alpha + \beta] + \frac{N-1}{2}Q_T[\alpha] + \frac{(\alpha + \beta)^2}{64\pi^2} \\ & \times \left(\ln \frac{\alpha + \beta}{\mu^2} - \frac{3}{2} \right) + \frac{(N-1)\alpha^2}{64\pi^2} \left(\ln \frac{\alpha}{\mu^2} - \frac{3}{2} \right). \end{aligned} \quad (63)$$

The gap equation for the sigma condensate in turn becomes

$$\sigma_0^2 \left[(\alpha + \beta)(\sigma_0^2) - \frac{4\lambda}{3N}\sigma_0^2 \right] = c. \quad (64)$$

From this equation one sees that the pion mass does not vanish even below the critical point

$$m_\pi^2 = \frac{c}{\sigma_0} \quad (65)$$

and the sigma-meson mass is given by

$$m_\sigma^2 = 2\sigma_0^2 \left(\frac{d(\alpha + \beta)}{d(\phi_c^2)} \Big|_{\sigma_c = \sigma_0, \pi_c = 0} - \frac{4\lambda}{3N} \right) + \frac{c}{\sigma_0}, \quad (66)$$

where the derivative term is the same form as eq. (42).

5.2 Numerical results

Here we show some numerical results of the above equations. We first concentrate only on the thermal effects and ignore the quantum corrections. This approximation is frequently used in the literature when one is interested in the thermal effects. We will include the quantum corrections in the next subsection. Throughout this and the next section, we set $N = 4$, *i.e.*, the $SU(2)_L \times SU(2)_R$ chiral symmetry which corresponds to the real world of one sigma and three pions.

5.2.1 Thermal corrections

We use the values at zero temperature as initial conditions of numerical calculation. Because we ignore the quantum corrections at this stage, the solutions of the SD-equations $\alpha + \beta$ and β are equal to m_σ and m_π at $T = 0$ as in the chiral limit. Initial parameters are determined from the condition at $T = 0$:

$$c = f_\pi m_\pi^2(T = 0), \quad (67)$$

$$\lambda = \frac{3}{f_\pi^2} (m_\sigma^2(T = 0) - m_\pi^2(T = 0)), \quad (68)$$

$$m^2 = -\frac{1}{2} m_\sigma^2(T = 0) + \frac{3}{2} m_\pi^2(T = 0), \quad (69)$$

where $f_\pi (= 93 \text{ MeV})$ is the pion decay constant at zero temperature.

For the mass of σ , Particle Data Group gives the value of 400–1200 MeV [21]. We take $m_\sigma = 600 \text{ MeV}$ as typical values. Qualitative properties change little if the mass of the sigma meson is to be $m_\sigma = 1 \text{ GeV}$. m^2 is negative because the spontaneous symmetry breaking takes place.

These results are essentially the same as ref. [13] because we ignore the quantum corrections here, so we do not give their results. In ref. [13], however, the author identifies $\alpha + \beta$ and β with the sigma-meson mass and the pion mass, respectively, so that he concludes that the NG theorem is violated at finite temperature. As discussed before, we do not regard these values as the physical meson masses and rather consider them simply as variational parameters. We calculate the sigma meson mass according to eq. (66) instead.

When we evaluate the sigma meson mass in the chiral limit in this approximation, however, we encounter a difficulty due to the infrared singularity. As shown in subsect. 3.2, there is a factor $P_2[x]$ in the equation of the sigma meson mass eq. (66). Its temperature-dependent part $P_{2,T}[x]$ has the form

$$\begin{aligned} P_{2,T}[x] &= \frac{1}{4\pi^2} \int_0^\infty dy \frac{y^2}{y^2 + \left(\frac{x}{T}\right)^2} \\ &\times \left[\left(1 + \frac{1}{\sqrt{y^2 + \left(\frac{x}{T}\right)^2}} \right) \frac{1}{e^{\sqrt{y^2 + \left(\frac{x}{T}\right)^2}} - 1} \right. \\ &\left. + \frac{1}{\left(e^{\sqrt{y^2 + \left(\frac{x}{T}\right)^2}} - 1 \right)^2} \right]. \end{aligned} \quad (70)$$

We naively suppose that this temperature-dependent part vanishes at $T \rightarrow 0$. This is indeed so for $x \neq 0$. However, this factor gives a non-zero contribution if x approaches to zero for $T \rightarrow 0$ keeping x/T finite. This phenomenon actually occurs in the strict chiral limit if we neglect quantum corrections, because one of the solutions of the SD-equation, α , vanishes at $T = 0$. The fact that α/T approaches to a non-zero value at $T = 0$ can be confirmed by numerical calculation. Thus the sigma-meson mass $m_\sigma(T \rightarrow 0)$ deviates from $m_\sigma(T = 0) = 600 \text{ MeV}$. We consider that this is because we have taken only the temperature dependent part in $P_2[x]$. In fact, in the chiral limit, the temperature-independent part $P_{2,Q}[x]$ is also infrared divergent at $T = 0$ and is to cancel the infrared singularity in $P_{2,T}[x]$. So it may not be possible to separate the temperature-dependent part and the temperature-independent one in $P_2[x]$.

There are two possibilities to solve this problem. One is to introduce the explicit chiral symmetry breaking term $-\sigma$, as seen above. This makes α finite at $T = 0$, corresponding to the finite-mass pions. So if we take the pion mass to be considerably large, *e.g.*, $m_\pi(T = 0) = 138 \text{ MeV}$, this problem apparently disappears. Another possibility, which we think is better, is to include quantum corrections, which makes α finite even in the chiral limit. The study along this line is given in the following subsection.

5.2.2 Thermal and quantum corrections

In this subsection, we include the quantum corrections following the regularization of the auxiliary field discussed in sect. 4. We have then another parameter μ which is introduced in the dimensional regularization and corresponds to the renormalization scale which is the regularization parameter in the present case. As μ is a free parameter, we must choose a suitable value μ at which the spontaneous symmetry breaking actually occurs. All the other parameters are determined at the chosen μ as reasonable values. Before proceeding to finite temperature, we perform a simple estimation in our model.

In QCD, the following relation is well satisfied:

$$f_\pi^2 m_\pi^2 = 2m_q \langle \bar{q}q \rangle, \quad (71)$$

where q denotes the light u, d quarks. This relation is called the Gell-Mann–Oakes–Renner (GMOR) relation and is derived from the low-energy theorem of QCD. In the chiral perturbation theory which is one of the low-energy effective models of QCD, this relation is also satisfied to the first order in the quark mass. This equation shows that the pion mass square is proportional to the quark mass. The deviation from it corresponds to higher-order effects of the chiral perturbation theory. Since the chiral perturbation theory is a perturbation expansion in both external momenta of the NG bosons and the current quark masses, if this relation is satisfied up to the relatively large quark mass, it means that the chiral perturbation theory is valid up to this momentum scale. We recently investigated the

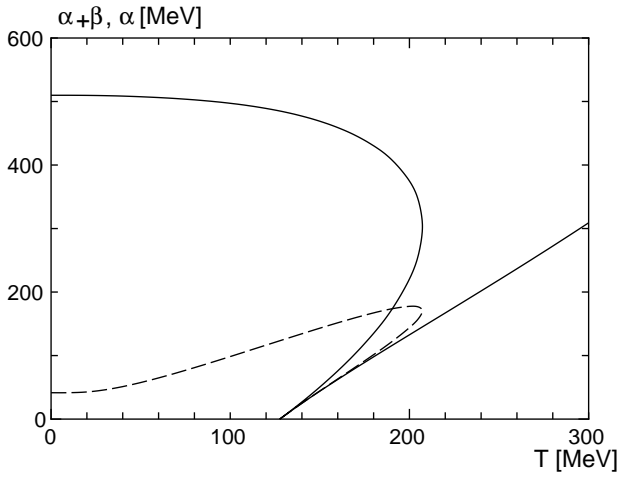


Fig. 3. The solutions of SD-equations for $m_\sigma(T=0) = 600$ MeV and $\mu = 320$ MeV. The solid line is $\alpha + \beta$ and the dashed one is α .

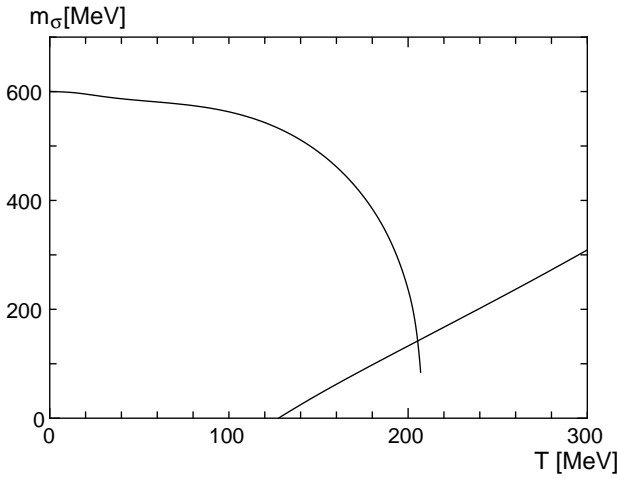


Fig. 4. m_σ for $m_\sigma(T=0) = 600$ MeV and $\mu = 320$ MeV.

behavior of the GMOR relation by constructing the pion as a relativistic bound state of quark and antiquark pair using the Bethe-Salpeter equation [22]. We showed that the GMOR relation is quite well satisfied at least up to the strange quark mass (~ 150 MeV) region.

In our linear sigma-model the relation corresponding to GMOR relation is

$$f_\pi m_\pi^2 = c. \quad (72)$$

We can evaluate the c -dependence of m_π^2 in the following way. First we determine the values of c and other parameters at $m_\pi = 138$ MeV. Then by changing the value of c with the other parameters fixed, we calculate $\alpha + \beta$ and α , and the meson masses. The obtained results are shown in fig. 6. We have confirmed that these results are almost independent of the parameter, μ . These three values show almost linear dependence on c . In the chiral limit ($c = 0$), the sigma condensate which corresponds to the pion decay constant is about 87 MeV. This is very similar

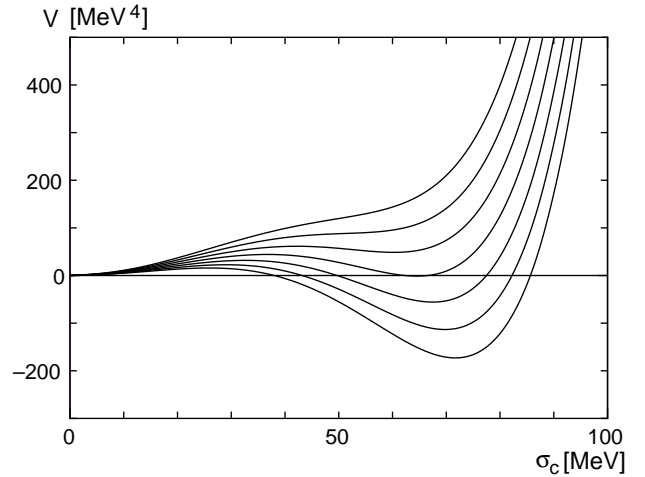


Fig. 5. Effective potential for $m_\sigma(T=0) = 600$ MeV and $\mu = 320$ MeV. Solid lines denote the case of $T = 198$ MeV, 194 MeV, ..., 174 MeV from top to bottom.

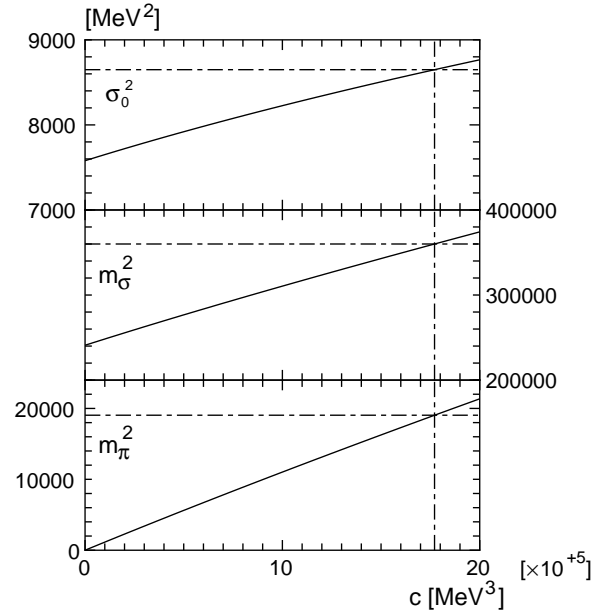


Fig. 6. σ_0^2 , m_σ^2 and m_π^2 for $\mu = 320$ MeV.

to the one obtained in the chiral perturbation theory [23] which gives 88 MeV. The sigma-meson is about 490 MeV (we have fixed $m_\sigma = 600$ MeV at $m_\pi = 138$ MeV) and the pion mass of course vanishes in the chiral limit. Thus we see that the linear sigma-model reproduces some results well obtained from the chiral perturbation theory.

Now we proceed to finite temperature. Once the quantum corrections are included, $\alpha + \beta$ and α do not agree with the sigma and pion masses even at $T = 0$. We need to determine them by solving the SD-equations and the gap equation for the sigma condensate. We show the results of the initial parameters obtained in this way in tables 1–3. The value T'_c is the point where the σ_0 curve crosses the temperature axis. We see that for smaller μ , α can be negative so that the effective potential becomes complex

Table 1. Initial Parameters for $T = 0$

$m_\pi = 0$ MeV						
m_σ [MeV]	μ [MeV]	λ	α [MeV]	$\alpha + \beta$ [MeV]	m^2 [MeV ²]	T'_c [MeV]
600	320	90.2	41.5	510	-122375	128
600	400	102	119	542	-94208	105
600	500	122	183	594	-47679	68.4
600	567	139	223	632	0	0
600	TREE	125	0	600	-180000	132
1000	410	156	29	670	-218034	130
1000	500	177	156	715	-128224	93.2
1000	599	208	239	774	0	0
1000	TREE	347	0	1000	-500000	132

Table 2. Initial Parameters for $T = 0$ (various values of μ)

$m_\pi = 138$ MeV						
m_σ [MeV]	μ [MeV]	λ	α [MeV]	$\alpha + \beta$ [MeV]	m^2 [MeV ²]	
600	220	66.9	55.9	460	-96324	
600	300	77.8	114	493	-83362	
600	400	94.7	165	540	-59939	
600	520	119	223	603	-5633	
600	TREE	118	138	600	-151434	
1000	380	142	54.8	655	-197043	
1000	400	146	87.5	664	-180329	
1000	500	172	184	718	-95776	
1000	580	196	245	765	-106	
1000	TREE	340	138	1000	-471434	

unphysical. For larger μ , m^2 becomes positive so that the spontaneous symmetry breaking does not occur. As a result a proper range of μ is naturally determined as is given in the table. Because the parameter μ is free within this range, we show as an example a result for $\mu = 320$ MeV in the case of $m_\sigma(T = 0) = 600$ MeV.

We show the temperature dependence of the solutions of the SD-equations, the physical meson masses and the effective potential in the chiral limit in figs. 3-5. Temperature dependence of the sigma condensate is also shown in fig. 9. The effective potential shows the signature of the first-order phase transition, whose behavior is similar to the case without quantum corrections.

We see also from the solutions of the SD-equations and the sigma condensate that the equations have two solutions at some temperature. This is a typical feature of the first-order phase transition, that is, the upper solution corresponds to the (local) minimum of the potential and the lower to the maximum of the potential.

Table 3. Initial Parameters for $T = 0$ (various pion masses)

$m_\sigma = 600$ MeV, $\mu = 320$ MeV				
m_π [MeV]	λ	α [MeV]	$\alpha + \beta$ [MeV]	m^2 [MeV ²]
0	90.2	41.5	510	-122375
20	89.8	44.7	509	-121078
60	87.8	64.8	507	-112237
100	84.6	93.5	504	-97658
138	80.9	125	502	-79760

As the temperature increases from zero, the sigma condensate decreases, jumps to zero at T_c and remains zero above T_c . Likewise, in the figures $\alpha + \beta$ (α) changes along the solid (dashed) line, jumps to the lower solid line at T_c and increases along this line.

If the deconfinement transition takes place at the same critical temperature as the chiral phase transition, there are free quarks and gluons for $T > T_c$. But recent lattice simulations suggest that there are some correlations of quarks and gluons above T_c [24]. An instanton liquid model also favors this result [25]. This may imply there exist hadronic excitations even above T_c .

The fact that the phase transition is the first order agrees with other mean-field approaches [5-7]. It is, however, generally believed mainly from the analysis of the renormalization group equation that the actual phase transition of the $O(4)$ linear sigma-model should be of the second order. We comment on this problem in the following section.

The critical temperature is about 184 MeV for $\mu = 320$ MeV and about 178 MeV for $\mu = 500$ MeV. For the case of $m_\sigma(T = 0) = 1$ GeV, it is about 220 MeV for $\mu = 400$ MeV and about 230 MeV for $\mu = 580$ MeV. So the parameter μ little affects the critical temperature. These values are reasonable since other approaches in the linear sigma model predict similar values.

We note that there is no peculiar behavior in the curves of the sigma-meson mass, shown in fig. 4. This is because α does not vanish even at $T = 0$ due to the quantum corrections, shown in fig. 3. On the other hand, the pions

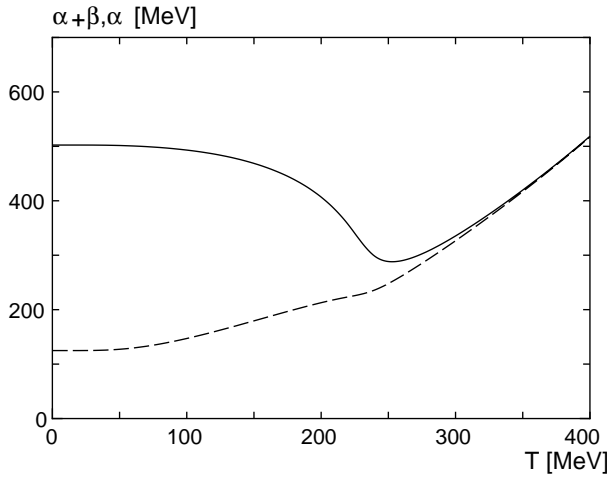


Fig. 7. The SD-solutions $m_\sigma(T=0) = 600$ MeV, $m_\pi(T=0) = 138$ MeV and $\mu = 320$ MeV. The solid line is $\alpha + \beta$ and the dashed one is α .

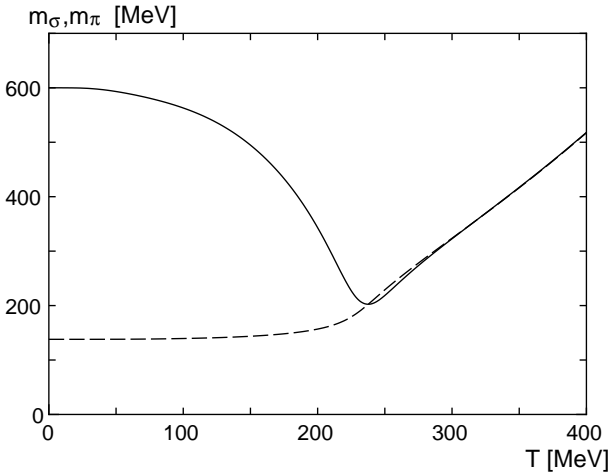


Fig. 8. m_σ for $m_\sigma(T=0) = 600$ MeV, $m_\pi(T=0) = 138$ MeV and $\mu = 320$ MeV.

remain massless below the critical temperature even in this case.

Finally we show the cases with the explicitly chiral symmetry breaking, *i.e.* $m_\pi(T=0) = 138$ MeV in figs. 7-9.

6 Comments on effects of other loops

We have so far calculated the physical quantities in the Hartree approximation. This means that the solutions of the SD-equations for the propagator, α and β , are momentum-independent. In the superdaisy diagrams, this assumption is quite natural because no external momentum enters into loops. In fact many authors use this approximation within the formalism of the superdaisy diagrams in the mean-field approach [5–7].

However, note that near the critical temperature there is no reason to believe that only the superdaisy diagrams

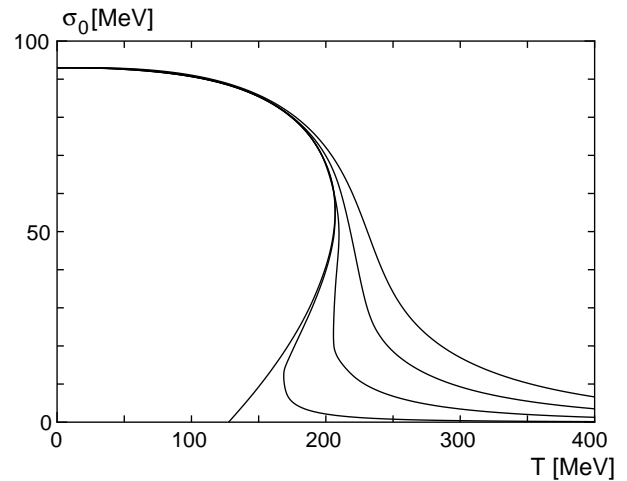


Fig. 9. Sigma condensates σ_0 for $m_\sigma(T=0) = 600$ MeV, $\mu = 320$ MeV and $m_\pi(T=0) = 0, 20, 60, 100, 138$ MeV from left to right.

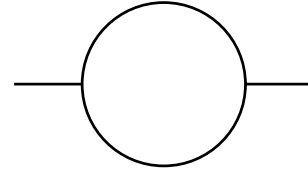


Fig. 10. The loop diagram contributing to the two-point Green function which comes from the setting-sun-type diagram.

are dominant. Within the framework of the Hartree approximation, the first-order phase transition takes place as in our case. (If we take the large- N limit in the CJT approach, the result is the second-order, see ref. [13].) But the analyses of the renormalization group equations etc. suggest that the phase transition of the $O(4)$ linear sigma-model is of the second order [26,27]. Arnold and Espinosa pointed out that other loop diagrams than the superdaisy diagrams are important near the critical temperature [28]. So the result for the order of the transition cannot be trusted in the calculation with the superdaisy diagrams only.

The next leading loop diagram contributing to the effective potential is a setting-sun-type diagram. This diagram contributes to the two-point Green function as in fig. 10.

It is obvious that the external momentum enters into the loop and therefore the self-energy depends on the external momentum. In our CJT formalism, this means that α and β become momentum-dependent. Furthermore, we may have to take into account the wave function renormalization by this diagram. As far as we know, there is no calculation in which this diagram is incorporated consistently. Here we briefly estimate the contribution of this setting-sun diagram to the order of the phase transition in the CJT approach by some approximation. The term of the effective potential corresponding to the setting-sun

diagram is given by

$$\begin{aligned}
 & -\frac{4\lambda^2}{9N^2}\phi_c^a\phi_c^c\int_{T,p}\int_{T,q}\{2G_{ad}(p)G_{bd}(q)G_{bc}(-p-q) \\
 & + G_{ac}(p)G_{bd}(q)G_{bd}(-p-q)\}. \quad (73)
 \end{aligned}$$

Hereafter we consider a simple case with $N = 1$, since we are now interested in the phase transition rather than the hadron phenomenology. The CJT effective potential is now given by

$$\begin{aligned}
 V(\phi_c, G) &= \frac{1}{2}m^2\phi_c^2 + \frac{\lambda}{24}\phi_c^4 + \frac{1}{2}\int_{T,k}\ln G^{-1}(k) \\
 &+ \frac{1}{2}\int_{T,k}\mathcal{D}^{-1}(k)G(k) + \frac{\lambda}{8}\left(\int_{T,k}G(k)\right)^2 \\
 &- \frac{\lambda^2}{12}\phi_c^2\int_{T,p}\int_{T,q}G(p)G(q)G(-p-q) \quad (74)
 \end{aligned}$$

with

$$\mathcal{D}^{-1}(k) = k^2 + m^2 + \frac{\lambda}{2}\phi_c^2. \quad (75)$$

From the extremal condition, we obtain the SD-equation for G :

$$\begin{aligned}
 G^{-1}(p) &= \mathcal{D}^{-1}(p) + \frac{\lambda}{2}\int_{T,k}G(k) \\
 &- \frac{\lambda^2}{2}\phi_c^2\int_{T,k}G(k)G(-p-k) \quad (76)
 \end{aligned}$$

Since the integral of the last term in eq. (76) depends on the external momentum, it is natural to put the form of G as

$$G(p) = \frac{A(p)}{p^2 + M(p)^2}. \quad (77)$$

Here we take a very simple form in which any external momentum dependence in the parameters are neglected, *i.e.*, $A = 1$ and $M(p) = M$. It can be calculated relatively easily in the CJT formalism by this procedure. Though this is rather strong approximation and nothing but a kind of the superdaisy approximation, we see that the order of the phase transition can change in some case.

We have calculated two cases that neglect quantum corrections. One corresponds to the small λ , $\lambda \sim O(1)$, and the other the large one, $\lambda \sim O(100)$. The latter is the order used in the low-energy effective model of the π, σ mesons. The results are as follows. Without the setting-sun diagram, if the coupling constant λ is small, the weak first-order phase transition occurs. We see that in this case the order becomes second by including the setting-sun (fig. 11), while in the strong-coupling region the order does not change. So the contribution of the setting-sun diagram in this approximation changes the order of the phase transition from the first to the second if the original first order is rather weak.

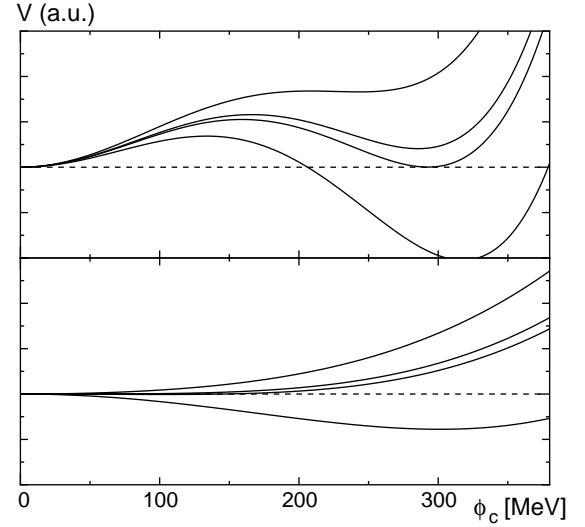


Fig. 11. Effective potentials near the critical temperature without (above) and with (below) the setting-sun ($\lambda = 1$). The values of the potential are in arbitrary units.

Chiku and Hatsuda calculated this diagram in the optimized perturbation theory [10]. They include the resummed mass parameter in the Lagrangian from the beginning and renormalize it first assuming that the mass parameter is a momentum-independent constant factor though the inclusion of this diagram makes the mass parameter momentum-dependent. We think that this corresponds to our approximation used above. Nevertheless, they pointed out that if this diagram is included in their formalism, the order of the phase transition may change from the first to the second.

Nachbagauer also calculated this diagram in the one-component ϕ^4 theory in the CJT action [29]. He calculated the self-energy in a certain approximation in order to get rid of the UV-divergence and evaluated the Green function in the Padé approximation. He also pointed out that this diagram is important near the critical temperature, although he does not evaluate the order of the transition.

In conclusion, it seems to be necessary to include the effects of the higher-order diagrams, if we discuss the behavior near the critical temperature. Further investigation is needed in the future.

7 Summary and conclusion

We have studied the $O(N)$ symmetric linear sigma-model at finite temperature in the Hartree approximation using the CJT effective potential. This model has been considered as the low-energy effective model of the low-energy mesons, *i.e.*, the sigma-meson and the pions. It has so far been understood that the NG theorem is not satisfied, or the pions are massive at finite temperature in the CJT formalism unless the large- N limit is taken. We showed that this is caused by an improper derivation of the effective potential, such as an $O(N)$ -violating form for the solution

of the SD-equation for the propagator. If the $O(N)$ symmetric form is chosen and one defines the meson masses as the second derivatives of the effective potential, the NG theorem is always satisfied even at finite values of N . This is also satisfied at any order of the loop expansion of Γ_2 , which is the contribution of the two-particle irreducible vacuum graphs.

We have also used a regularization of the CJT effective potential from the analogy to renormalization of the auxiliary field. This works apparently both in the symmetry broken and restored phases, while the conventional renormalization works only in the symmetry restored phase.

When one is interested in effects of the thermal contributions at finite temperature, quantum corrections are often neglected. The classical equation of motions are satisfied at $T = 0$ in this case and the thermal effects are added as temperature increases. We have solved the gap equation for the sigma condensate, the SD-equations and the meson masses numerically in this approximation. As a result the first-order phase transition occurs, because this corresponds to the mean-field approach. Related to this approximation, we encounter a problem in calculating the sigma-meson mass in the chiral limit because of the infrared singularity. This difficulty is removed if one includes the quantum corrections because the solutions of the SD-equations which cause the infrared singularity remain finite in the chiral limit at $T = 0$. So consistent treatments including the quantum corrections which is discussed in sect. 4 are needed to obtain more realistic results. In subsect. 5.2.2, based on the above notion, we incorporated the quantum corrections. As a result the infrared singularity disappeared and the behavior apparently became better for both the chiral limit and the finite pion mass.

The resulting phase transition in the $O(4)$ linear sigma-model is of the first order, which is consistent with other mean-field approaches. However the second-order phase transition is reported in this model in the renormalization group equation analyses. In fact near the critical temperature other loop contributions may become significant and we showed the simple estimation of the effects of the setting-sun diagram in the CJT approach. As a result, the order of the phase transition can change with this inclusion. Therefore further investigation is needed if one studies the order of the phase transition more rigorously.

Appendix A. The $O(N)$ linear sigma-model with auxiliary fields

We applied the technique of renormalization of auxiliary fields for the CJT approach in the main text. In this appendix we analyze the $O(N)$ linear sigma-model at finite temperature by using auxiliary fields in order to show a similarity with the CJT approach. We also point out that if we analyze the $O(N)$ linear sigma-model at finite temperature with the auxiliary fields from the beginning, some difficulties associated with the NG theorem arise again. Such an approach was carried out in ref. [30], where one auxiliary field was introduced. However, this causes the

violation of the NG theorem unless some 1-loop terms are added in the sigma- and pion self-energy. We require a more general form of auxiliary fields which do not the $O(N)$ symmetry in order to relate it to the CJT approach.

For the original Lagrangian, after shifting the fields $\phi^a \rightarrow \phi^a + \phi_c^a$, we introduce an auxiliary field with an $N \times N$ symmetric matrix:

$$\begin{aligned} & \exp \left\{ - \int d^4x \frac{\lambda_0}{6N} (\phi^2)^2 \right\} \\ &= \int \mathcal{D}\chi \exp \left\{ - \int d^4x \left[\frac{\lambda_0}{6N} (\phi^2)^2 - \frac{3N}{8\lambda_0(N+8)} \right. \right. \\ & \quad \left. \left. \times \left(\chi^{ab} - \frac{2\lambda_0}{3N} \phi^2 \delta^{ab} - \frac{4\lambda_0}{3N} \phi^a \phi^b \right)^2 \right] \right\}. \end{aligned} \quad (\text{A.1})$$

We take the saddle point approximation for χ^{ab} , *i.e.*, $\chi^{ab} \approx \chi_c^{ab}$, and then the effective potential at the 1-loop order is obtained:

$$\begin{aligned} V(\phi_c, \chi_c) &= \frac{m^2}{2} \phi_c^2 + \frac{\lambda}{6N} (\phi_c^2)^2 - \frac{3N}{8\lambda(N+8)} \chi_c^2 \\ &+ \frac{1}{2} \int_{T,k} \ln \det \left\{ \left(k^2 + m^2 + \frac{2\lambda}{3N} \phi_c^2 + \frac{\chi_c^{cc}}{N+8} \right) \delta^{ab} \right. \\ & \left. + \frac{4\lambda}{3N} \phi_c^a \phi_c^b + \frac{2}{N+8} \chi_c^{ab} \right\}. \end{aligned} \quad (\text{A.2})$$

From the extremal conditions for χ_c^{ab} and ϕ_c^a , we obtain the following equations:

$$\begin{aligned} M_1^2 &= m^2 + \frac{2\lambda}{3N} + \frac{2\lambda}{3} \frac{1}{N+8} \\ & \times \int_{T,k} \left\{ \frac{N+3}{k^2 + M_1^2} + \frac{(N+4)/N}{k^2 + M_2^2} \right\}, \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} M_2^2 &= m^2 + \frac{2\lambda}{N} + \frac{2\lambda}{3} \frac{1}{N} \\ & \times \int_{T,k} \left\{ \frac{(N+4)(N-1)/(N+8)}{k^2 + M_1^2} + \frac{1}{k^2 + M_2^2} \right\}, \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} & \phi_c^2 \left\{ m^2 + \frac{2\lambda}{3N} \phi_c^2 + \frac{2\lambda}{N} \right. \\ & \left. \times \int_{T,k} \frac{1}{k^2 + M_2^2} + \frac{2\lambda(N-1)}{3N} \int_{T,k} \frac{1}{k^2 + M_1^2} \right\} = 0, \end{aligned} \quad (\text{A.5})$$

where we defined α and β by $\chi_c^{ab} = \alpha \delta^{ab} + \beta \frac{\phi_c^a \phi_c^b}{\phi_c^2}$ without loss of generality of χ_c^{ab} and put

$$M_1^2 = m^2 + \frac{2\lambda}{3N} \phi_c^2 + \frac{(N+2)\alpha + \beta}{N+8}, \quad (\text{A.6})$$

$$M_2^2 = m^2 + \frac{2\lambda}{N} \phi_c^2 + \frac{(N+2)\alpha + 3\beta}{N+8}. \quad (\text{A.7})$$

Equations (A.3), (A.4) and (A.5) are similar to eqs. (31), (32) and (37) in the CJT approach, respectively. In fact

the renormalized form of the effective potential (A.2) can be obtained by requiring renormalization conditions for M_1 and M_2 at zero temperature, which means the renormalization of the auxiliary fields. Our regularization in the CJT approach is based on this idea.

A difficulty occurs, however, when we consider the case of finite temperature. Our numerical calculation shows that the eqs. (25), (A.4) and (A.5) do not have any physical solutions at finite temperature in contrast to the CJT approach. This means that the summation of the superdaisy diagrams does not work well and we may require some additional terms to recover it as in ref. [30]. The CJT approach, on the other hand, does not have such a problem, though the forms of the equations are similar. We think that the CJT case corresponds to special auxiliary fields other than those of this section, but the relations between them remain as the future problem.

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